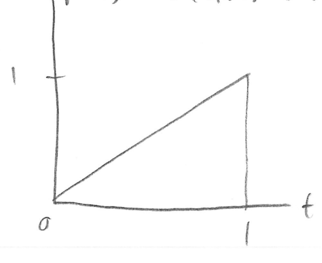
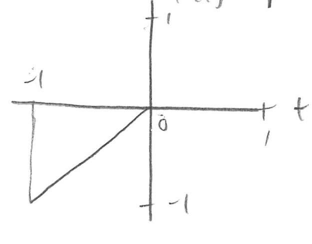
1.a

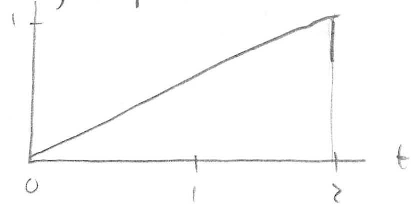
$p(t) = t(u(t) - u(t-1))$



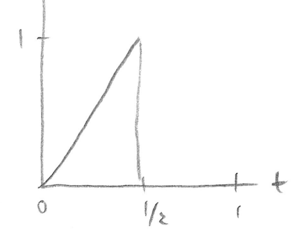
$f(t) = -p(-t)$



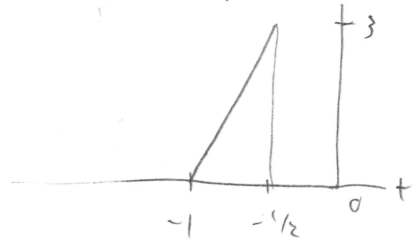
$g(t) = p(\frac{t}{2})$



$h(t) = p(2t)$

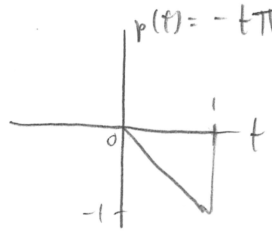


$m(t) = 3p(2(t+1))$

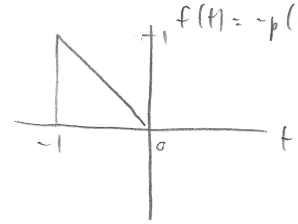


other definition:

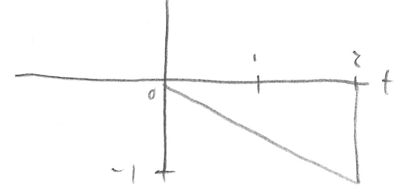
$p(t) = -t\Pi(t - \frac{1}{2})$



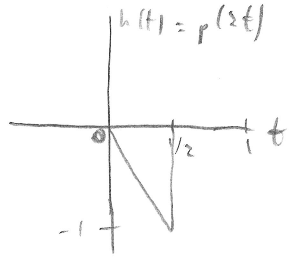
$f(t) = -p(-t)$



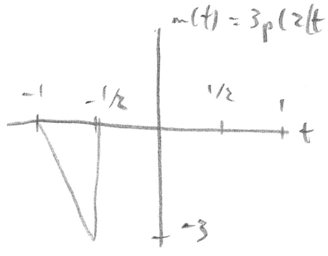
$g(t) = p(t/2)$



$h(t) = p(2t)$



$m(t) = 3p(2(t+1))$



1.b.

$$E\{p\} = \int_{-\infty}^{\infty} |p(t)|^2 dt = \int_0^1 t^2 dt = \frac{t^3}{3}\Big]_{t=0}^{1} = \frac{1}{3}(1-0) = \boxed{\frac{1}{3}}$$

$$E\{-p\} = \int_{-\infty}^{\infty} |-p(t)|^2 dt = \int_{-\infty}^{\infty} |p(t)|^2 dt = E\{p\}$$ (the typo will not affect the energy)

$$E\{f\} = \int_{-\infty}^{\infty} |-p(-t)|^2 dt = \int_{-\infty}^{\infty} |p(-t)|^2 dt = \int_{-\infty}^{\infty} |p(u)|^2 du = E\{p\} = \boxed{1/3}$$

$(u = -t)$

$$E\{p(at)\} = \int_{-\infty}^{\infty} |p(at)|^2 dt = \int_{-\infty/a}^{\infty/a} |p(u)|^2 \frac{du}{a} = \begin{cases} \frac{1}{a} E\{p\} & a > 0 \\ -\frac{1}{a} E\{p\} & a < 0 \end{cases} = \frac{1}{|a|} E\{p\}$$

$(a \in \mathbb{R})$ $(u = at)$

1.b. (cont)

So, $E\{p(at)\} = \frac{1}{|a|}E\{p(t)\}$, then $E\{g\} = 2E\{p\} = \boxed{\frac{2}{3}}$

and $E\{h\} = \frac{1}{2}E\{p\} = \boxed{\frac{1}{6}}$

$$E\{ap(t)\} = \int_{-\infty}^{\infty} |ap(t)|^2\,dt = |a|^2\int_{-\infty}^{\infty}|p(t)|^2\,dt = |a|^2 E\{p\}$$

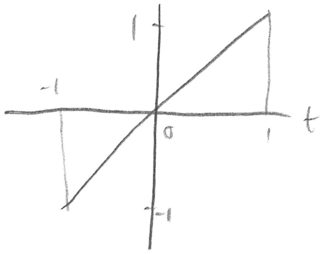
$$E\{p(t-t_0)\} = \int_{-\infty}^{\infty}|p(t-t_0)|^2\,dt = \int_{-\infty+t_0}^{\infty+t_0}|p(u)|^2\,du = \int_{-\infty}^{\infty}|p(u)|^2\,du = E\{p\}$$

$$u = t - t_0$$

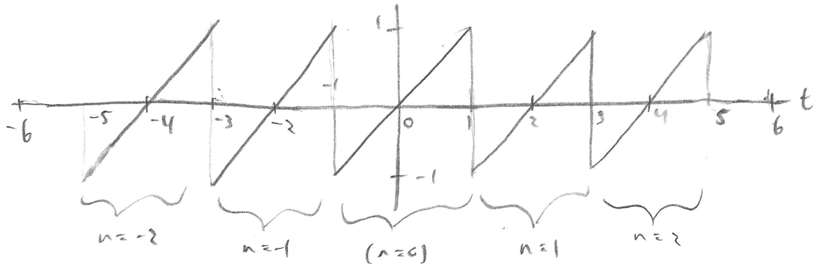
$$E\{m\} = E\{3p(2(t+1))\} = 9E\{p(2(t+1))\} = 9E\{p(2t)\} = \frac{9}{2}E\{p\} = \frac{9}{6} = \boxed{\frac{3}{2}}$$

1.c

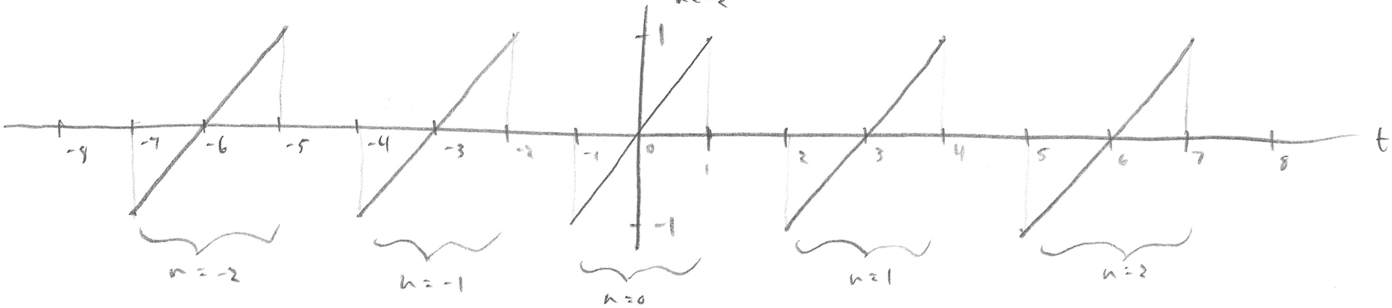
$s(t) = p(t) - p(-t)$



$x(t) = \sum_{n=-2}^{2} s(t-2n)$

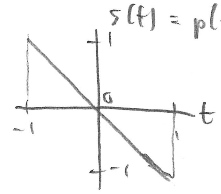


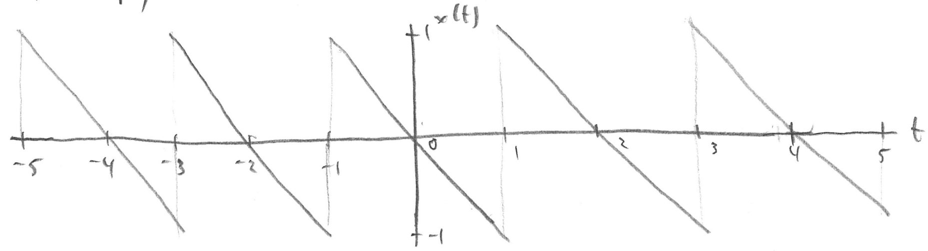
$y(t) = \sum_{n=-2}^{2} s(t-3n)$

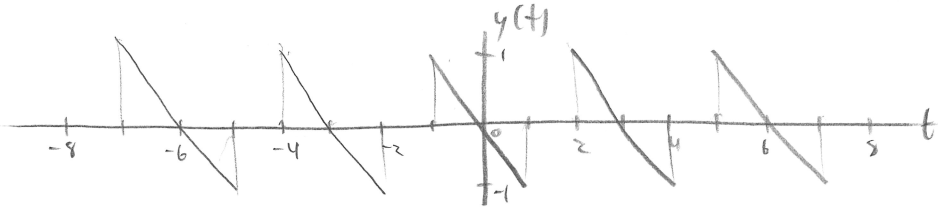


other definition of $p(t)$ is simply reflected about the t-axis.

$s(t) = p(t) - p(-t)$







$\boxed{1.d}$

Let x_1 and x_2 be non-overlapping signals, i.e. $x_1 = 0$ if $x_2 \neq 0$ and $x_2 = 0$ if $x_1 \neq 0$. Note that we can still have $x_1 = x_2 = 0$. We can break up the Energy Integral into pieces where only one of the signals is non-zero:

$$\int_{-\infty}^{\infty} |x_1(t) + x_2(t)|^2 dt = \int_a^b |x_1(t) + 0|^2 dt + \int_b^c |0 + x_2|^2 dt + \int_c^d |x_1 + 0|^2 dt + \cdots$$

then we end up with: $E\{x_1 + x_2\} = E\{x_1\} + E\{x_2\}$

For completeness:

$$|x_1 + x_2|^2 = (x_1 + x_2)(x_1 + x_2)^* = (x_1 + x_2)(x_1^* + x_2^*) = x_1 x_1^* + x_1 x_2^* + x_2 x_1^* + x_2 x_2^* =$$

$$= |x_1|^2 + |x_2|^2 + x_1 x_2^* + x_2 x_1^* = |x_1|^2 + |x_2|^2 + 0 + 0 \text{ (because they're non-overlapping)}$$

$$\int_{-\infty}^{\infty} |x_1 + x_2|^2 dt = \int_{-\infty}^{\infty} |x_1|^2 + |x_2|^2 dt = \int_{-\infty}^{\infty} |x_1|^2 dt + \int_{-\infty}^{\infty} |x_2|^2 dt$$

Clearly, s, x, and y are all sums of nonoverlapping shifted versions of $p(t)$, so we can just sum the energies, remember that a time shift does not change energy.

$$E\{s\} = E\{p\} + E\{p\} = \boxed{\frac{2}{3}}$$

$$E\{x\} = (2N+1)\, E\{s\} = \boxed{\frac{2(2N+1)}{3}}$$

$$E\{y\} = \boxed{\frac{2(2N+1)}{3}}$$

$\boxed{2.a.}$

$$E\{2je^{-3|t|}\} = |2j|^2 E\{e^{-3|t|}\} = \frac{4}{|-3|} E\{e^{-|t|}\} = \boxed{\frac{4}{3}}$$

$\boxed{\text{energy signal}}$

$$E\{e^{-|t|}\} = \int_{-\infty}^{\infty} |e^{-|t|}|^2 dt = 2\int_0^{\infty} |e^{-t}|^2 dt \quad (\text{even integrand})$$

$$= 2\int_0^{\infty} e^{-2t} dt = 2 \left.\frac{e^{-2t}}{-2}\right]_{t=0}^{\infty} = 0-(-1) = 1$$

$$P\{2je^{-3|t|}\} = \boxed{0} \text{ because it's an energy signal}$$

$$E(T) = \int_{-T}^{T} |x|^2 dt \le E\{x\}$$

$$\lim_{T\to\infty} E(T) = E\{x\}$$

$$P\{x\} = \lim_{T\to\infty} \frac{1}{2T} E(T) \le \lim_{T\to\infty} \frac{E}{2T} = E \lim_{T\to\infty} \frac{1}{2T} = 0$$

$\boxed{2.b}$

$$E\{2je^{(-4+2j)t}u(-t)\} = 4\int_{-\infty}^{0} |e^{(-4+2j)t}|^2 dt = 4\int_{-\infty}^{0} e^{-8t} dt$$

$$|e^{j\omega t}|^2 = 1$$

$$= -\frac{4}{8} \left. e^{-8t}\right]_{t=-\infty}^{0} = -\frac{1}{2}(1-\infty) = \boxed{\infty} \quad \text{not an energy signal}$$

$$P\{2je^{-4t}e^{2jt}\} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{0} |2je^{-4t}e^{2jt}|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\left(\left.-\frac{1}{2}e^{-8t}\right]_{t=-T}^{0}\right)$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left(\frac{1}{2}(e^{+8T}-1)\right) = \boxed{\infty} \quad \text{not a power signal}$$

$\boxed{2.c}$

$$E\{3e^{-|t|}\cos(2t)\} = 9\int_{-\infty}^{\infty} |e^{-|t|}\cos(2t)|^2 dt = 18\int_0^{\infty} e^{-2t}\cos^2(2t)\,dt$$

even integrand

$$= \frac{18}{2}\int_0^{\infty} e^{-u}\cos^2(u)\,du = 9\cdot\frac{1}{4}\int_0^{\infty} e^{-u}(e^{ju}+e^{-ju})^2 du = \frac{9}{4}\int_0^{\infty} e^{-u}(e^{2ju}+e^{-2ju}+2)\,du$$

$u = 2t$

$$= \frac{9}{4}\left(\frac{e^{(-1+2j)u}}{-1+2j} + \frac{e^{(-1-2j)u}}{-1-2j} + 2e^{-u}\right]_{u=0}^{\infty} = \frac{9}{4}\left(\frac{0-1}{-1+2j} + \frac{0-1}{-1-2j} + 0-2\right)$$

$$= \frac{9}{4}\left(2 - \frac{-1-2j+-1+2j}{|-1+2j|^2}\right) = \frac{9}{4}\left(2+\frac{2}{5}\right) = \boxed{\frac{27}{5}} \quad \text{energy signal} \quad P\{3e^{-|t|}\cos(2t)\} = \boxed{0}$$

$\boxed{2.d.}$

$$E\{5je^{-3jt}\} = 25\int_{-\infty}^{\infty}|e^{-3jt}|^2\,dt = 25\int_{-\infty}^{\infty}1\,dt = \boxed{\infty}$$ not an energy signal

$$P\{5je^{-3jt}\} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|5je^{-3jt}|^2\,dt = \lim_{T\to\infty}\frac{25}{2T}\int_{-T}^{T}dt = \lim_{T\to\infty}25 = \boxed{25}$$

Power signal

$\boxed{3.a}$

$$\int_{-\infty}^{\infty}x(t)\delta(t)\,dt = x(0) \rightarrow \int_{-\infty}^{\infty}x(t)\delta(t-t_0) = x(t_0)$$

$$\int_{-\infty}^{\infty}e^{-t^3}\delta(t-3)\,dt = \boxed{e^{-3^3}}$$

$\boxed{3.b}$

$$\int_{-\infty}^{\infty}\cos(5t^2)\delta(2t-3)\,dt = \int_{-\infty}^{\infty}\cos\left(5\left(\tfrac{u}{2}\right)^2\right)\delta(u-3)\frac{du}{2} = \boxed{\tfrac{1}{2}\cos\left(5\left(\tfrac{3}{2}\right)^2\right) = \tfrac{1}{2}\cos\left(\tfrac{45}{4}\right)}$$

$u = 2t$

$du = 2dt$

$t = u/2$

$\boxed{3.c}$

$$\int_{-\infty}^{5/3}\cos(5t^2)\delta(2t-3)\,dt \qquad 2t-3=0 \Rightarrow t = \tfrac{3}{2} < \tfrac{5}{3}$$

$$= \boxed{\tfrac{1}{2}\cos\left(\tfrac{45}{4}\right)}$$

$\boxed{3.d}$

$$\int_{t}^{\infty}\tau^2\delta(\tau-1)\,d\tau = \boxed{\begin{cases} 0 & t>1 \\ \text{undef.} & t=1 \\ 1 & t<1 \end{cases}}$$

$\boxed{4.}$

$$\mathcal{E} = \int_{-\infty}^{\infty}|\sin(\tfrac{\pi}{2}t)p(t) - ctp(t)|^2\,dt = \int_0^1|\sin(\tfrac{\pi}{2}t) - ct|^2\,dt$$

$$= \int_0^1\left(\sin^2(\tfrac{\pi}{2}t) - 2ct\sin(\tfrac{\pi}{2}t) + c^2t^2\right)dt = \tfrac{1}{2} + -\frac{8c}{\pi^2} + \tfrac{1}{3}c^2$$

$$\frac{d\mathcal{E}}{dc} = \tfrac{2}{3}c - \frac{8}{\pi^2} = 0 \rightarrow c = \boxed{\frac{12}{\pi^2}}$$

$\boxed{\text{4 cont.}}$

using orthogonality instead:

$$c = \frac{\langle x, \phi \rangle}{\|\phi\|^2} = \frac{\int_{-\infty}^{\infty} \sin(\frac{\pi}{2}t)\, p(t)\, t\, p(t)\, dt}{\int_{-\infty}^{\infty} t^2 p^2(t)\, dt} = \frac{\int_0^1 t \sin(\frac{\pi}{2}t)\, dt}{\int_0^1 t^2\, dt} = \frac{4/\pi^2}{1/3} = \boxed{\frac{12}{\pi^2}}$$

$\boxed{\text{5.a}}$

$T = 10$

$$\langle \phi_n, \phi_m \rangle = \frac{1}{T}\int_0^T e^{2\pi j n t/T} e^{-2\pi j m t/T}\, dt = \frac{1}{T}\int_0^T e^{2\pi j (n-m) t/T}\, dt$$

$$= \frac{1}{T}\left(\frac{e^{2\pi j (n-m)} - 1}{2\pi j (n-m)} \right) \quad (m \neq n)$$

$$= \frac{1}{T}\left(\frac{1 - 1}{2\pi j (n-m)} \right) \quad (n - m \in \mathbb{Z})$$

$$= 0$$

$$= \frac{1}{T}\int_0^T dt \quad (n = m)$$

$$= 1$$

$$\langle \phi_n, \phi_m \rangle = \boxed{\begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases} = \delta[n-m]}$$

$\boxed{\text{5.b}}$

$$\langle x, \phi_n \rangle = \frac{1}{10}\int_0^{10} \cos(7\pi t)\, \phi_n(t)\, dt = \frac{1}{10}\int_0^{10} \frac{1}{2}\left(e^{2\pi j 35 t/10} + e^{2\pi j (-35) t/10}\right) \phi_n(t)\, dt$$

$$= \frac{1}{10}\int_0^{10} \frac{1}{2}\phi_{35}(t)\, \phi_n(t)\, dt + \frac{1}{10}\int_0^{10} \frac{1}{2}\phi_{-35}(t)\, \phi_n(t)\, dt$$

$$\boxed{= \begin{cases} \frac{1}{2} & |n| = 35 \\ 0 & \text{else} \end{cases} = \frac{1}{2}\delta[n-35] + \frac{1}{2}\delta[n+35]}$$

$\boxed{\text{5.c}}$

$$\sum_{n=-\infty}^{\infty} |X[n]|^2 = \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}$$

HW 1

$\boxed{5.d}$

$$y = 5\sin(9\pi t - \tfrac{\pi}{4}) = \frac{5}{2j}\left(e^{2\pi j\, 45 t/10} e^{-j\frac{\pi}{4}} - e^{2\pi j(-45)t/10} e^{+\frac{\pi}{4}j}\right)$$

$$= \frac{5}{2j} e^{-j\pi/4} \phi_{45}(t) - \frac{5}{2j} e^{+j\pi/4} \phi_{-45}(t)$$

$$\boxed{\langle y, \phi_n\rangle = \begin{cases} \frac{5}{2j} e^{-j\frac{\pi}{4}} & n = 45 \\ -\frac{5}{2j} e^{+j\frac{\pi}{4}} & n = -45 \\ 0 & \text{else} \end{cases} = \frac{5}{2j} e^{-j\frac{\pi}{4}} \delta[n-45] - \frac{5}{2j} e^{+j\frac{\pi}{4}} \delta[n+45]}$$

$\boxed{5.e}$

$$\sum_{n=-\infty}^{\infty} |y_n|^2 = \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = \boxed{\frac{25}{2}}$$

$\boxed{5.f}$

$$\langle a\phi_n + b\phi_m, a\phi_n + b\phi_m\rangle = \langle a\phi_n, a\phi_n\rangle + \langle a\phi_n, b\phi_m\rangle + \langle b\phi_m, a\phi_n\rangle + \langle b\phi_m, b\phi_m\rangle$$

$$= |a|^2\langle \phi_n, \phi_n\rangle + ab^*\langle \phi_n, \phi_m\rangle + ba^*\langle \phi_m, \phi_n\rangle + |b|^2\langle \phi_m, \phi_m\rangle$$

$$= |a|^2 \|\phi_n\|^2 + |b|^2 \|\phi_m\|^2 + 2\,\mathrm{Re}(ab^*\langle \phi_n, \phi_m\rangle)$$

$$= |a|^2 + |b|^2 \quad \text{if } m \neq n$$

$$\langle x, x\rangle = \left|\tfrac{1}{2}\right|^2 + \left|\tfrac{1}{2}\right|^2 = \boxed{\tfrac{1}{2}} = E\{x\}$$

$$\langle y, y\rangle = \left|\frac{5}{2j} e^{\pi/4 j}\right|^2 + \left|-\frac{5}{2j} e^{-\pi/4 j}\right|^2 = \boxed{\frac{25}{2}} = E\{y\}$$

$$\langle x, y\rangle = \langle a\phi_{35} + a\phi_{35}, b\phi_{45} - b^*\phi_{-45}\rangle = \langle a\phi_{35}, b\phi_{45}\rangle + \langle a\phi_{35}, -b^*\phi_{-45}\rangle$$
$$+ \langle a\phi_{35}, b\phi_{45}\rangle + \langle a\phi_{-35}, -b^*\phi_{-45}\rangle$$

$$= \boxed{0} \quad \text{(all terms are orthogonal)}$$